MTH 161 - Fall 2013

Lecture 1

1.1 Functions and their representations :

A function f is a rule that assigns to each element x in D exactly one element called , $f(x)$ in a set E.

[We generally work with functions where D and E are a set of real numbers, $\mathbb{R}$]

$\mathbb{N}$

$D \equiv$ domain , $E \equiv$ codomain $\mathbb{Z}$

$\mathbb{Q}$

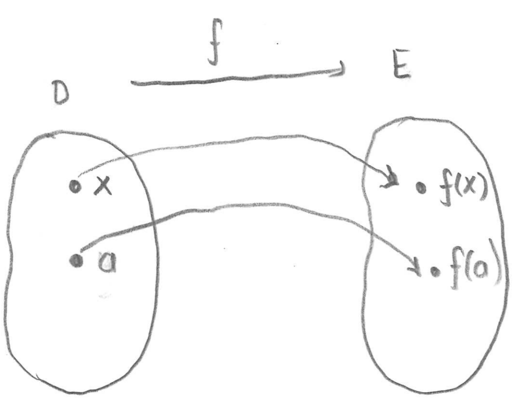
The number $f(x)$ is value of at x , called f of x .

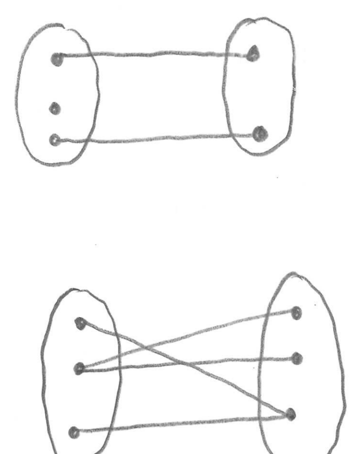
Range $\equiv$ set of all possible values of $f(x)$

[Think of it as a machine when x, an element in the domain enter , an element $f(x)$ spits out , based on the machines own rules . So domain $\equiv$ all possible inputs and range $\equiv$ all possible outputs] .

NOT A FUNCTION

ARROW DIAGRAMS



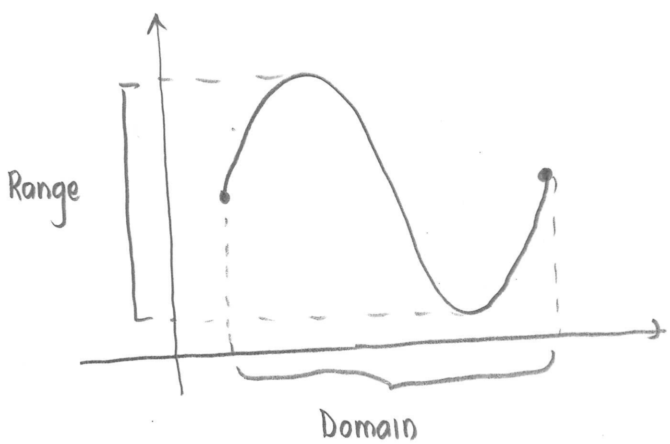


The way we are going to represent functions :

- graphically (via a graph)
- algebraically (via an explicit formula)
- * numerically (by a table of values)

GRAPHICALLY If f is a function with domain D, then its graph is the set of of ordered pairs $\{(x, f(x)) \mid x \in D\}$.

Simply when $D = E \subset \mathbb{R}$, the graph of f consists of all points (x, y) in the coordinate plane such that $y = f(x)$ and x is in the domain of f.



$f(x) = x^2$ [Algebraically]

When given an algebraic expression, we would like to determine the domain of the function.

Ex Find the domain of

a) $f(x) = \sqrt{x-2}$.

Because the square root of a negative is not defined (as a real number) the domain consists of all numbers such that $x-2 \geq 0$.

So this is the same as saying $x \geq 2$.

This can be expressed as $[2, \infty)$

b) $g(x) = \dfrac{1}{x^2-4x+3}$

- Since we are not allowed to divide by 0, we know that $g(x)$ is not defined when the denominator is 0.

So when is denominator $= 0$ i.e $x^2-4x+3 = 0$

$(x-1)(x-3) = 0 \Rightarrow x = 1, x = 3$

So domain is $x \neq 1$, $x \neq 3$

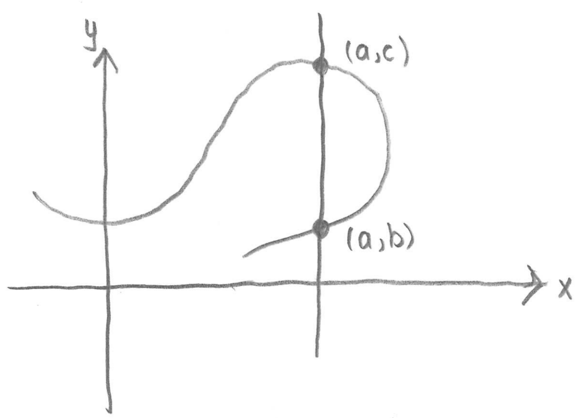
In interval notation, $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

[QUES We Know that a function can be represented as a curve in the xy-plane. So how do you determine when curves in the xy-plane are graphs of functions]

The vertical line test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Ex



This curve does not represent a function because a function cannot assign two values to a.

PIECEWISE DEFINED FUNCTIONS :

The functions are defined by different formulas in different parts of their domains.

Example 1 :

$$f(x) = \begin{cases} x , & x \leq -1 \\ 1 , & -1 < x \leq 2 \\ x+1 , & x > 2 \end{cases}$$

Find,

$f(-2)$, $f(0)$, $f(3)$ and sketch the graph.

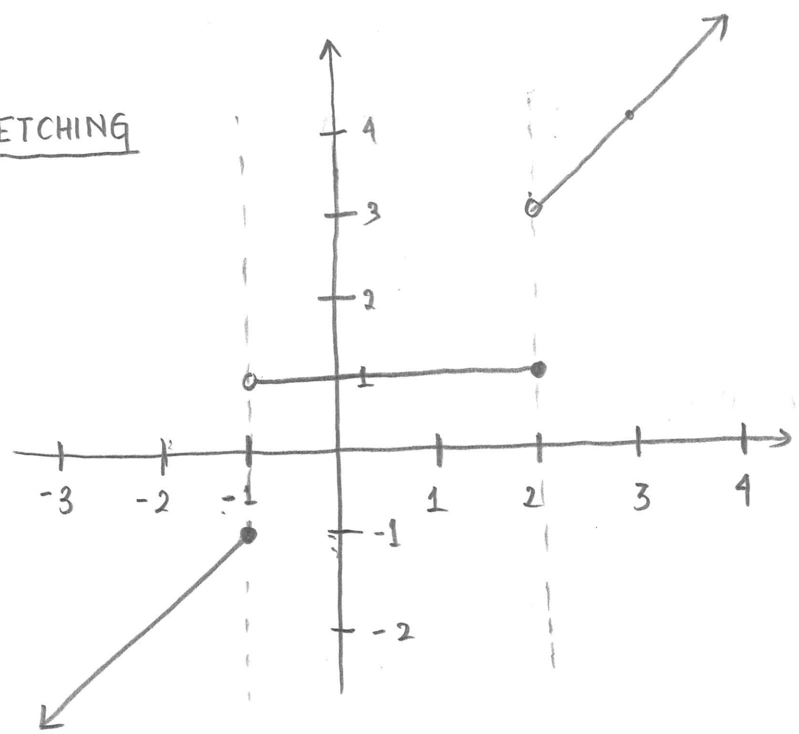
Ans Look at the value of the input x.

If $x \leq -1$, then $f(x) = x$. Hence, $f(-2) = -2$

Since $-1 < 0 \leq 2$, $f(0) = 1$

Finally, since $3 > 2$, $f(3) = 3+1 = 4$

SKETCHING



Example 2 :

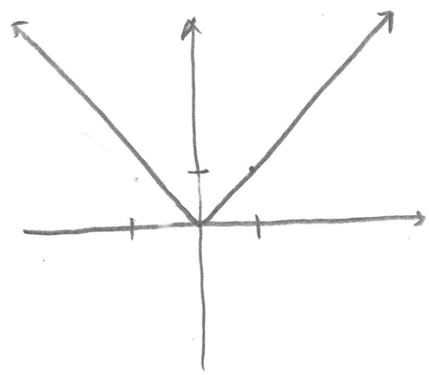
$f(x) = |x|$

How does it work ?

$|-1| = 1$, $|2| = 2$, $|7.6| = 7.6$

In general, it can be defined as a piecewise function :

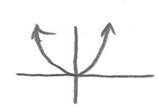
$$|x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$



SYMMETRY

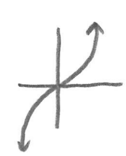
Even function : If for every x in the domain of f, the function satisfies $f(x) = f(-x)$, f is called an even function .

The geometric significance is that the graph is symmetric with respect to the y-axis .



Odd function : $f(-x) = -f(x)$

Symmetric with respect to the origin



LECTURE 1

Let $g(x) = x^4 + x^2 + 3$

Then we want to compute $g(-x)$

$$g(-x) = (-x)^4 + (-x)^2 + 3$$
$$= x^4 + x^2 + 3 \quad = g(x)$$

So g is an even function $\boxed{\text{EVEN}}$

$$h(x) = x^3 + x$$

$$h(-x) = (-x)^3 + (-x) \quad ; \quad -h(x) = -(x^3 + x)$$
$$= -x^3 - x \qquad\qquad = -x^3 - x$$

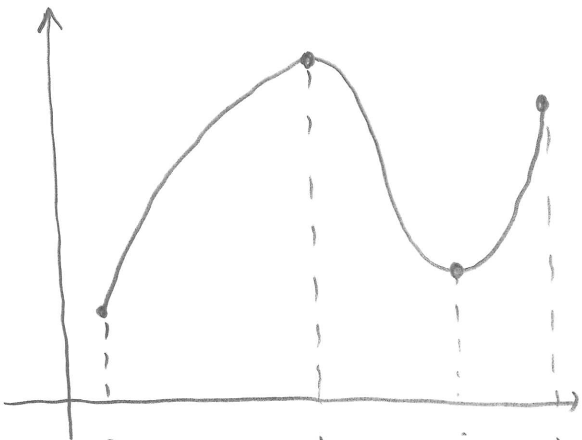
$$h(-x) = -h(x) \qquad \boxed{\text{ODD}}$$

$$f(x) = x^2 + x \qquad -f(x) = -(x^2 + x)$$
$$= -x^2 - x$$

$$f(-x) = (-x)^2 + (-x)$$
$$= x^2 - x$$

$\boxed{\text{NEITHER}}$

Increasing and Decreasing functions



Increasing : $[a,b]$, $[c,d]$

Decreasing : $[b,c]$

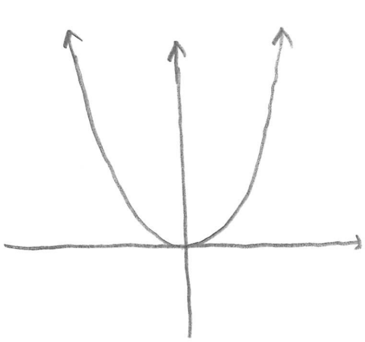
A function is called increasing on an interval I if

$f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I

called decreasing on I if

$f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I

Ex



Increasing : $[0,\infty)$

Decreasing : $(-\infty,0]$